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Physics-362

12/16/2015

**TITLE: FINAL PROJECT-FRACTALS, JULIA AND MANDELBROT SETS**

**Abstract:**

The study of fractals are a fascinating subject to study upon. This paper and presentation delves upon the examples of profound patterns of fractals throughout nature, and also looks at examples of application of fractals in science and technology. For the project I have written several samples of code written in Matlab that iterate fractals for the sake of demonstrating the pattern of fractals. The code are variations of Julia and Mandelbrot sets.

Keywords: fractal, self-similar, nature, application, recursion

**Introduction:**

Fractals:

Fractals are a rough geometric shape and phenomenon that exhibits a repeating pattern at virtually every scale. This phenomenon is also known as a self-similar pattern. Fractals have been a fascinating subject for me to look upon as a final project as they are virtually everywhere in nature, and in application of science and technology is profound.

**Body:**

Recursion:

All type and forms of fractals follow a mathematically recursive pattern. Recursion is a process in which something repeats itself in a self-similar way. All recursive patterns can be defined by two properties in Mathematics/Computer-Science:

1. Base Case(s): Starting Point.
2. Set of conditions that reduce all other cases to the base case(s).

The Fibonacci sequence is a well-known example of recursion.

For the Fibonacci sequence:

Pseudocode:

F(0)=0; //as base case 1;

F(1)=1; //as base case 2;

For(n>1){

F(n)=F(n-1)+F(n-2);

}

**[Barnsley et al. 1993]**

Mandelbrot sets:

Mandelbrot sets are a set of complex numbers c for which sequences

C, C2 + C, (C2 + C)2+C,((C2 + C)2 + C)2 +C................

Recursively, but doesn't approach infinity.

The Mandelbrot set can be represented by the polynomial

Z(0)=Z , Z(n+1)=Z(n)\*Z(n)+Z, n= 0,1,2,3,…......... **[Alfred 1998]**

Julia Sets:

Julia set is the complement of the Mandelbrot set. For every Mandelbrot set there is a Julia set in every point if one were zoom in on the Mandelbrot set **[Dickson n.d]**.

Fractals in Nature:

Fractal patterns are found all over the place throughout nature, and there are many examples of phenomenon exhibiting a fractal pattern.

Coastlines:

Natural terrain such as beaches and coastlines are neither shape or of a generic shape, but can be approximated as fractal in shape **[Barnsley et al. 1993]**.

Mountains/Plate-Tectonics:

As a result of tectonic plates colliding with one another, parts of the Earth's crust pushes upward by the path of least resistance resulting in terrain formations such as mountains, and volcanoes. Mountains when observed from above demonstrates neither a shape non-regular in pattern but that of a fractal pattern **[Barnsley et al. 1993]**.

Volcanoes/Plate-tectonics:

Due to tectonic plates diverging or converging, rifts form from the Earth's crust comes gushing with lava, ash, and smoke debris. This is known as a volcanic eruption that forms volcanoes. Volcanic eruptions from ruptures on the Earth's crust results in fragmentations of the crust that are neither of a generic shape but that of a self-similar pattern **[Gusev 2014]**.

Canyons:

As a result of over a long periods of time from soil erosion. The process of soil erosion is a gradual and repetitive process of rain, wind every year. Ravines form from a plateau resulting in a formation we call Canyons that are fractal in shape **[Turcotte 1991]**.

Spiral Shells:

In shells, the repeating spiral shape of seashells demonstrates a fractal pattern that is ever expanding as the shell spirals outward [**Fowler et al. 1992]**.

Plants:

The Plant Kingdom happens to have a lot of species that have a self-similar or fractal pattern in them. Tree branches are a result of one large tree trunk diverging into smaller trunks which are known as tree branches. The diverging point of the tree trunks are known as nodes, the branches can be considered internodes. For leaves in that of say ferns, tends to have self-similar patterns at different scales as the stem of the fern diverges and expands, the leaves also follow the same pattern. Broccoli plants demonstrate this self-similar feature where its flower buds grow and expand **[Prusinkiewicz et al. 2004]**.

Applications of Fractals:

Art:

Fractals happen to have many applications in real life. Art is by far the most obvious one. Fractals patterns are used to make beautiful works of art throughout the ages. Example of fractal patterns are the use of mosaics as a format to render works of art. Tiles are also another example on walls and on floors in which self-similar patterns are depicted for aesthetic purposes **[Fractal Art]**.

Fractal Antennas:

In many portable devices such as handhelds, cellphones, tablets, and laptops for instance; the shape of antenna used for wireless communication take a fractal shape. The antenna elements are arranged in a fractal array in all directions. The self-similar shape of the antenna is what allows the antenna to function invariant of antenna frequency and bandwidth. This fractal antenna is what makes wireless communication possible despite the shape and form factors of devices such as cell-phones, handhelds, tablets, and laptops, etc. **[Hohlfeld et al. 1999]**.

Suspension Cables:

In Civil-engineering, the use of steel cables in modern suspension bridges are what allow suspension bridges to span greater distances and greater payload, In this case the steel cables that suspend the bridge are consisted of repetitive strands of cables wrapped in a spiral increases the tensile strength of the cable. This self-similar steel cables is what allows the steel cables to bear greater stress loads than what would not be feasible with non-spiral steel cables under the same constraints **[Fractal Applications]**.

Computer Graphics:

In computer graphics the use of fractals in graphical algorithms generate greater samples of smaller polygons out of subdividing larger polygons by tiling, this effect is known as Tessellation. The greater amount of sampling helps improves the smoothness of the surface of the rendered object. The greater detail as a result of tessellation improves photorealism. Tessellation has a benefit reducing the needed amount of memory and memory bandwidth of a GPU (Graphics Processing Unit) needed to render highly detailed features **[Tariq 2009].** A GPU is a an integrated circuit that runs an wide array of ALUs and FPUs (Arithmetic Logic Units and Floating Point Units) optimized for processing binary data in parallel whereas a CPU (Central Processing Unit) processes data in a highly serial manner.

Camouflage Uniforms:

The use of camouflage for military applications have been a means to conceal military personnel and equipment from a hostile adversary since the inception of modern rifles of the late 19th century. In the recent decades the use of fractal algorithms in graphics programs help produce digitally rendered combat uniforms that provides greater concealment (and in turn greater survivability) to their respective terrain than prior uniforms adopted by military personnel; by a process that is dubbed “C2G” or “Camouflage Designated Enhanced Fractal Geometry” in military jargon**[O’Neill 2005].**

**Conclusion:**

Fractals are a rough geometric shape and phenomenon that exhibits a repeating pattern at virtually every scale. This phenomenon is also known as a self-similar pattern. Fractal patterns have been found almost everywhere in nature, and have profound applications in art, science and technology alike. Fractals have been a fascinating subject for me to study as a final research project topic.

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